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The Crystal and Molecular Structures of Strontium Tartrate Trihydrate and Calcium Tartrate Tetrahydrate*

By G.K.Ambady

Centre of Advanced Study in Biophysics and Crystallography, University of Madras, Madras 25, India

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Strontium tartrate trihydrate belongs to the monoclinic system with space group P_{2_1} . The structure of this compound was solved from a three-dimensional beta synthesis phased on the strontium atom and refined by the full-matrix least-squares method. Calcium tartrate tetrahydrate belongs to the orthorhombic system with the space group $P_{2_12_12_1}$. The structure of this compound was solved by three-dimensional electron density maps and least-squares refinements. The final *R* indices (for observed structure factors) are 0.109 for strontium tartrate and 0.103 for calcium tartrate. The structures are stabilized by a three-dimensional network of hydrogen bonds. The tartrate in behaves like a chelating agent towards strontium and calcium. An interesting feature in the structure of calcium tartrate is the large dihedral angle (22°) which the C–O(H) bond makes with the plane of the neighbouring carboxyl group. In both structures the cation exhibits an 8-fold coordination.

Introduction

The determination of the crystal structures of strontium and calcium tartrates forms part of an investigation on the conformation of the tartrate ion in the crystalline state. The first compound to be studied in this series was sodium tartrate dihydrate (Ambady & Kartha, 1968).

Experimental

Strontium tartrate trihydrate

Transparent plate-like crystals of $SrC_4H_4O_6.3H_2O$ were obtained from an aqueous solution of the salt at room temperature. One crystal was chosen and ground into a needle shape with nearly uniform cross section. The crystal data are given below.

| $SrC_4H_4O_6.3H_2O;$ | M = 289.6 |
|--------------------------------|---------------------------------|
| $a = 7.55 \pm 0.02$ Å | $b = 10.06 \pm 0.02$ Å |
| $c = 6.47 \pm 0.02 \text{ Å}$ | $\beta = 102^{\circ}0' \pm 10'$ |
| Z=2 | $D_x = 2.003 \text{ g.cm}^{-3}$ |
| D_m (by flotation) 2 | 2.028 g.cm ⁻³ |
| $\mu(Cu K\alpha) = 82 cm^{-1}$ | 1 |

Space group $P2_1$ (from systematic absences).

Calcium tartrate tetrahydrate

When equal volumes of 0.2N solutions of calcium chloride and Rochelle salt were mixed and kept at room temperature, needle shaped crystals of the tetrahydrate made their appearence (Chattaway, 1916).[†] The presence of calcium in the crystals was confirmed by flame photometer analysis. The crystal data obtained by the author together with the values reported in a previous determination (Evans, 1936) are given below.

- - - -

CaC₄H₄O₆.4H₂O;
$$M = 260 \cdot 1$$

Present author Evans
 $a = 9 \cdot 24 \pm 0 \cdot 02$ Å $9 \cdot 20$ Å
 $b = 10 \cdot 63 \pm 0 \cdot 02$ $10 \cdot 54$
 $c = 9 \cdot 66 \pm 0 \cdot 02$ $9 \cdot 62$
 $Z = 4$ 4
 $D_x = 1 \cdot 821$ g.cm⁻³
 $D_m = 1 \cdot 818$ g.cm⁻³ $1 \cdot 848$ g.cm⁻³
 μ (Cu K α) = 67 cm⁻¹

Space group $P2_12_12_1$ (from systematic absences).

The cell dimensions of both compounds were determined from precession photographs.

Collection of intensity data

Three-dimensional intensity data were collected by the multiple film equi-inclination Weissenberg technique using nickel-filtered copper radiation. Total numbers of reflexions recorded were 887 for strontium tartrate and 903 for calcium tartrate. The intensities were estimated visually, corrected for Lorentz and polarization factors and placed on an absolute scale by Wilson's method. The strontium tartrate data were corrected for absorption assuming the crystal to be a cylinder. Absorption correction was not applied to the calcium tartrate data as μr in this case was less than unity. Initial inter-layer scaling was achieved with the help of the respective cross layer data.

Determination of the structures

Strontium tartrate trihydrate

The x and z coordinates of the strontium atom were readily obtained from the Lp inverse sharpened Patterson function computed with the h0l intensity data.

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The y coordinate was assumed to be 0.25. The next step was to compute the h0l and hk0 beta synthesis (Ramachandran & Raman, 1959). However, both syntheses proved difficult to interpret because of the serious overlap in the h0l map and doubling of peaks due to a spurious mirror in the hk0 map. The structure was ultimately solved from a three-dimensional beta synthesis. This type of electron density Fourier synthesis has been successfully applied in the solution of a more complicated structure in this laboratory, namely, that of L-arginine hydrobromide, monohydrate (Mazumdar & Srinivasan, 1966). The beta synthesis uses coefficients of the type

$$(|F_o|^2/|F_{\mathfrak{Sr}}|) \exp i\alpha_{\mathfrak{Sr}}$$

where $|F_{Sr}|$ and α_{Sr} are the amplitude and the phase of the structure factor contribution from the heavy atom. Actually the synthesis in its weighted form was computed (Ramachandran & Ramachandra Ayyar, 1963). The weighted coefficients may be expressed as

 $W^2(|F_o|^2/|F_{\rm Sr}|) \exp i\alpha_{\rm Sr}$

where

$$W = \tanh X$$
 (Woolfson, 1956)

$$X = |F_o| \cdot |F_{\rm Sr}| / \sum_{i=1}^{Q} f_i^2$$
.

The denominator represents the sum of the scattering factors of all the light atoms. Since there was only one heavy atom in the asymmetric unit, a spurious centre of inversion occurred in the map, resulting in the duplication of peaks. There were also a few strong spurious peaks but practically all of them were located close to the strontium peak. It was possible, however, to locate all but one atom from this map, the average peak heights of oxygen and carbon atoms being 7 e.Å^{-3} and 6 e.Å^{-3} respectively. The remaining atom [in the water molecule O(8)] was located in the second electron density Fourier synthesis computed with phases calculated from twelve out of the thirteen atoms of the asymmetric unit.

In the structure obtained from the beta synthesis, it was observed that the tartrate ion was in the L-configuration. For the sake of uniformity the configuration was changed to the D-form. A solution of the crystals used in this study in dilute hydrochloric acid was found to be dextrorotatory, thereby showing that the compcund is the D-tartrate. Examination of the hk0 intensity data recorded in a Nonius integrating goniometer revealed a few Bijvoet pairs with measurable intensity differences. In all such cases the observed and calculated values of the ratio I_H/I_H were nearly the same, thus agreeing with the classical determination of the absolute configuration of tartaric acid by Bijvoet, Peerdeman & van Bommel (1951).

Calcium tartrate tetrahydrate

The coordinates of the calcium atom were initially obtained from the h0l and 0kl Lp inverse sharpened

Patterson projections, and later more accurately from a 3-D Patterson synthesis. The new values were

$$x = y = z = 0.433$$

Minimum functions plotted on 0kl and h0l projections did not yield any new information and consequently attempts to solve the structure from projections were abandoned. A 3-D electron density Fourier synthesis ρ_1 phased on the calcium atom was computed but it was not possible to identify the tartrate ion, as the map contained a large number of peaks of height above $2 e A^{-3}$. The strategy finally adopted was to sandwich least-squares refinements between successive electron density Fourier syntheses. The thermal parameter of an incorrectly placed atom invariably increased with successive least-squares cycles (Rossmann, Jacobson, Hirshfeld & Lipscomb, 1959). Such atoms are thereafter omitted from succeeding electron density Fourier syntheses. In the present analysis seven of the strongest peaks appearing in ϱ_1 were selected as possible atomic sites and their coordinates were refined through three cycles of least squares. At the end of the third cycle the thermal parameters of two of the atoms had shot up to 6.2 Å² and 5.4 Å² from the initial value of $2 \cdot 3 \text{ Å}^2$, while in the case of the rest of the atoms, the thermal parameters were below 3.5 Å². These two atoms were removed and a second electron density Fourier synthesis ρ_2 was computed with the phases from calcium and the remaining five light atoms, the latter being treated as nitrogen atoms. The synthesis revealed the positions of all but two atoms (water molecules) of the asymmetric unit. An electron density difference Fourier synthesis revealed the positions of these two water oxygens also.

Refinement of the structures

The two structures were refined by the full-matrix least-squares method using the ORFLS program of Busing, Martin & Levy (1962), the atoms being ascribed individual isotropic thermal parameters. A unit weighting scheme was employed, the unobserved F_o 's being given zero weight. Further, a dozen reflexions which showed large discrepancies (due to extinction, misindexing etc.) were omitted from the final cycles of refinement. The shifts of the parameters of calcium tartrate were about a tenth of the estimated standard deviations. But in the case of strontium tartrate refinement was stopped when the shifts of the parameters were about half the estimated standard deviations. It was also observed that the thermal parameters of some of the atoms in the structure of strontium tartrate assumed rather low values. This effect could be due to under-correction of the data for absorption (Werner, 1965). Correction based on cylindrical approximation was not satisfactory because of the irregular cross section of the crystal. However, it was better than no correction. Although it is well known that the unit weighting scheme is not suitable for least-squares refinement of photographic data, no further refinements of these two structures were attempted with more appropriate weighting functions, primarily because the two structures appeared to have refined to a reasonable degree of accuracy. The final R indices for strontium tartrate and calcium tartrate were 0.109 and 0.103 respectively. The lists of final observed and calculated structure factors for strontium and calcium tartrates are given in Tables 1 and 2 respectively. The final (fractional) coordinates of the atoms with the estimated standard deviations are given in Tables 3 and 4. The intramolecular bond distances are given in Table 5 and the bond angles in Table 6.

The atomic scattering factors for Sr^{2+} given by Thomas & Umeda (1957) were used after including the anomalous dispersion correction for the real part. The atomic scattering factors for O, C and Ca²⁺ were taken from *International Tables for X-ray Crystallo*graphy (1962).

Table 1. Observed and calculated structure factors for strontium D-tartrate

The columns give l, $10|F_o|$, $10|F_c|$ in that order.

The scale factors for layers k running from zero to eight are 1.01, 1.03, 1.04, 1.06, 1.03, 1.03, 1.04, 1.01 and 1.02.

Reflexions marked * were omitted from the final refinement.

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Table 2. Observed and calculated structure factors for calcium tartrate tetrahydrate

The columns give l, $10|F_o|$, $10|F_c|$ in that order. The scale factors for the layers k running from zero to eight are 0.95, 0.84, 1.21, 0.92, 0.90, 0.91, 0.93, 0.88 and 0.96.

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| 340 236 249 244 105 230 0 10 206 | , | 318 223 246 245 76 247 1 104 | 6 5 10 11 | 105 10 151 10 77 0 1 6 1 | | 55 14 53 59 0 1 22 05 4 59 0 1 22 59 0 1 22 59 0 1 22 50 5 50 5 50 5 50 5 50 5 50 5 50 5 5 | 35 12 2 pc 55 2 2 55 2 3 69 0 496 1 377 2 255 3 295 5 4 5 165 | 35 12 2.50 4.29 53 2 2 1 69 0 4.96 4.29 1 3.77 364 2 255 243 3 2.93 309 05 4 4.28 4.72 5 165 1.70 | 15 12 230 447 2 1 4 13 2 2 1 4 4 4 4 1 377 364 8 7 3 7 364 8 7 3 3 293 309 4 4 4 4 7 7 5 165 170 0 1 | $\begin{array}{cccccccccccccccccccccccccccccccccccc$ | $\begin{array}{cccccccccccccccccccccccccccccccccccc$ | $\begin{array}{c ccccccccccccccccccccccccccccccccccc$ | $\begin{array}{cccccccccccccccccccccccccccccccccccc$ | $\begin{array}{c ccccccccccccccccccccccccccccccccccc$ | $\begin{array}{c ccccccccccccccccccccccccccccccccccc$ | $\begin{array}{c ccccccccccccccccccccccccccccccccccc$ | $\begin{array}{c ccccccccccccccccccccccccccccccccccc$ | $\begin{array}{c ccccccccccccccccccccccccccccccccccc$ | $\begin{array}{c ccccccccccccccccccccccccccccccccccc$ | $\begin{array}{c ccccccccccccccccccccccccccccccccccc$ | $ \begin{array}{cccccccccccccccccccccccccccccccccccc$ | $ \begin{array}{c ccccccccccccccccccccccccccccccccccc$ | $ \begin{array}{cccccccccccccccccccccccccccccccccccc$ | $ \begin{array}{cccccccccccccccccccccccccccccccccccc$ | $ \begin{array}{c ccccccccccccccccccccccccccccccccccc$ | $ \begin{array}{c ccccccccccccccccccccccccccccccccccc$ | $ \begin{array}{c ccccccccccccccccccccccccccccccccccc$ | $ \begin{array}{c ccccccccccccccccccccccccccccccccccc$ | $ \begin{array}{cccccccccccccccccccccccccccccccccccc$ | $ \begin{array}{cccccccccccccccccccccccccccccccccccc$ | $ \begin{array}{c ccccccccccccccccccccccccccccccccccc$ |

Table 3. Final coordinates (fractional) of atoms in strontium tartrate trihydrate, with mean standard deviations

| | x | У | Z | В |
|-------|--------|--------|--------|--------|
| Sr | 0.2724 | 0.7500 | 0.1566 | 0·91 Ų |
| O(1) | 0.4813 | 0.5495 | 1.0823 | 3.09 |
| O(2) | 0.7381 | 0.4849 | 0.9896 | 1.67 |
| O(3) | 0.4928 | 0.7922 | 0.9084 | 2.28 |
| O(4) | 0.4220 | 0.5340 | 0.5414 | 2.08 |
| O(5) | 0.5358 | 0.8613 | 0.4075 | 2.59 |
| O(6) | 0.8199 | 0.8236 | 0.5681 | 1.93 |
| O(7)W | 0.1073 | 0.6747 | 0.7869 | 3.45 |
| O(8)W | 1.0017 | 0.6146 | 0.2384 | 6.10 |
| O(9)W | 0.0620 | 0.9277 | 0.2499 | 5.60 |
| C(1) | 0.6131 | 0.5687 | 0.9913 | 0.84 |
| C(2) | 0.6257 | 0.7013 | 0.8696 | 1.03 |
| C(3) | 0.6075 | 0.6787 | 0.6344 | 1.31 |
| C(4) | 0.6544 | 0.7951 | 0.5220 | 0.96 |

Average e.s.d. for strontium, 0.001 Å; for oxygen and carbon, 0.02 Å; for the water oxygen atoms O(8) and O(9), 0.03 Å.

Table 4. Final coordinates (fractional) of atoms in calcium tartrate tetrahydrate, with mean standard deviations

| | x | У | Z | В |
|-----------------------|--------|---------|--------|---------|
| Ca | 0.4363 | 0.4275 | 0.4328 | 0·95 Å2 |
| O(1) | 0.2661 | 0.2588 | 0.4805 | 1.94 |
| O(2) | 0.2023 | 0.0593 | 0.5139 | 2.15 |
| O(3) | 0.5251 | 0.2021 | 0.3904 | 1.65 |
| O(4) | 0.4112 | -0.0346 | 0.2778 | 1.41 |
| O(5) | 0.7131 | -0.0332 | 0.5079 | 1.67 |
| O(6) | 0.6939 | -0.0780 | 0.2810 | 2.00 |
| O (7) <i>W</i> | 0.1784 | 0.1762 | 0.1659 | 3.30 |
| O(8)W | 0.9524 | 0.0802 | 0.3126 | 3.74 |
| O(9)W | 0.8168 | 0.3099 | 0.1749 | 4.37 |
| O(10)W | 0.5242 | 0.1576 | 0.0856 | 2.05 |
| C(1) | 0.2962 | 0.1422 | 0.4879 | 1.38 |
| C(2) | 0.4528 | 0.1094 | 0.4705 | 1.08 |
| C(3) | 0.4804 | -0.0201 | 0.4091 | 2.05 |
| C(4) | 0.6401 | -0.0449 | 0.4011 | 1.56 |

 $\sigma(x) = \sigma(y) = \sigma(z)$ for Ca, 0.002 Å; for other atoms, 0.01 Å.

| Table : | 5. Intramole | cular bond | l distances | in s | strontium |
|---------|--------------|------------|-------------|------|-----------|
| | and | calcium t | artrates | | |

| | Sr tartrate | Ca tartrate |
|--------------------|-------------------|---------------------|
| C(1)-O(1) | 1·27 ± 0·03 Å | 1.27 ± 0.014 Å |
| C(1) - O(2) | 1.27 | 1.26 |
| C(4) - O(5) | 1.25 | 1.24 |
| C(4)–O(6) | 1.23 | 1.31 |
| C(2)-O(3) | 1.42 | 1.44 |
| C(3)–O(4) | 1.45 | 1.43 |
| C(1) - C(2) | 1.56 | 1.50 |
| C(2) - C(3) | 1.54 | 1.52 |
| C(3) - C(4) | 1.54 | 1.50 |
| Distances betwe | een non-bonded at | oms in the molecule |
| | Sr tartrate | Ca tartrate |
| $O(1) \cdots O(3)$ | 2·70 Å | 2·61 Å |
| $O(4) \cdots O(6)$ | 2.65 | 2.65 |
| $O(1) \cdots O(2)$ | 2.24 | 2.22 |
| $O(5) \cdots O(6)$ | 2.21 | 2.25 |
| | | |
| | | • |

Discussion

The tartrate ion

The intramolecular features of the tartrate ion as found in the two structures are shown in Figs. 1 and 2.

 Table 6. Intramolecular bond angles in strontium and calcium tartrates

| | Sr tartrate | Ca tartrate |
|--------------------|---------------------|---------------------|
| O(1)-C(1)-O(2) | $124 \pm 2^{\circ}$ | $123 \pm 1^{\circ}$ |
| O(1) - C(1) - C(2) | 116 | 116 |
| O(2) - C(1) - C(2) | 120 | 122 |
| C(1) - C(2) - O(3) | 110 | 110 |
| C(1) - C(2) - C(3) | 108 | 115 |
| O(3) - C(2) - C(3) | 113 | 111 |
| C(2) - C(3) - O(4) | 110 | 112 |
| C(2)-C(3)-C(4) | 108 | 110 |
| O(4) - C(3) - C(4) | 108 | 112 |
| C(3) - C(4) - O(5) | 113 | 118 |
| C(3) - C(4) - O(6) | 121 | 118 |
| O(5)-C(4)-O(6) | 125 | 124 |

The conformation of the tartrate ion is similar to that found in other tartrates (Beevers & Hughes, 1941; Stern & Beevers, 1950; Sadanaga, 1950; Parry, 1951; Sprenkels, 1956; van Bommel & Bijvoet, 1958; Ambady & Kartha, 1968). The two halves of the molecule, each consisting of a carboxyl group, a tetrahedral carbon and a hydroxyl oxygen, are individually planar. The two planes are so oriented that the four carbon atoms assume a zigzag planar configuration. One of the C-O distances in the carboxyl group is as great as 1.31 Å, suggesting that the carboxyl oxygen atom O(6) may be protonated. However, we suspect this distance to be an artifact arising from the inappropriate weighting scheme; for the same reason the standard deviations listed in Tables 4, 5 and 6 may not necessarily be the true values.

Both strontium and calcium are involved in the formation of planar bidentate chelate rings of the type MO(1)C(1)C(2)O(3) where M stands for the cation.



Fig. 1. Intramolecular features of the tartrate ion in strontium tartrate.

Details of the least-squares planes through different groups of atoms in the two structures and the deviations of the atoms from these planes are given in Table 7.

The carboxyl groups

In strontium tartrate neither carboxyl group is found to exhibit any large torsion angle about the C-C bond (Fig. 3). The carboxyl oxygen atoms O(1) and O(2) are involved in strong hydrogen bonds and further O(1) and O(3) take part in the formation of a planar fivemembered chelate ring. The planarity of the group C(2)O(3)C(1)O(1)O(2) may be partly dictated by the chelate ring and also by the fact that the hydrogen bond $O(9)-H\cdots O(2)$ lies almost in the plane of C(1)C(2)O(3).



Fig. 2. Intramolecular features of the tartrate ion in calcium tartrate.

The other carboxyl oxygen atoms O(6) and O(5) are hydrogen bonded to O(4A) and O(7A) respectively, but these bonds are so directed that they do not exert any torque on the carboxyl group. Hence this group remains nearly co-planar with C(4)C(3)O(4).

In calcium tartrate all four carboxyl oxygen atoms lie in proximity to either water molecules or hydroxyl groups at distances which suggest hydrogen bond formation. The environments of the two carboxyl groups viewed down C(1)-C(2) and C(4)-C(3) are shown in Fig.4. The angle between the planes defined by C(1)O(1)O(2) and C(1)C(2)O(3) is nearly 22°. As a result the angle between least-squares planes fitted on to the groups C(2)O(3)C(1)O(1)O(2) and C(3)O(4)C(4)-O(5)O(6) is nearly 90° instead of the usual value of about 60° found in tartrates. The large rotation of this carboxyl group about C(1)-C(2) appears to be due to the attractive influence of the cation on O(1) and also due to the hydroxyl group O(3B) of another molecule. Even though the distance between O(2) and O(4A) is 2.77 Å, it is unlikely that they are linked by a hydrogen bond since the angle C(3A)-O(4)-O(3) is nearly 167°. The forces acting on the other carboxyl group are relatively weak and moreover mutually opposing. One should therefore expect this group to be coplanar with C(4)C(3)O(4).

Molecular packing and hydrogen bonding: strontium tartrate trihydrate

The structure projected down the b axis is shown in Fig. 5. The backbone carbon atoms lie in sheets parallel to the bc plane. As in the structure of D-tartaric acid (Stern & Beevers, 1950) the carboxyl groups of the molecules related by symmetry are turned away from the screw axis.

There are two schemes of hydrogen bonding in the structure. A system of bonds running parallel to the b axis links the carboxyl oxygen atoms of one molecule with the hydroxyl groups of another molecule related by symmetry. The hydroxyl oxygen O(3*A*) is situated 2.77 and 2.59 Å away from O(1) and O(2) of the car-

| Table 7. Deviations of atoms from least-squares plane | ?* (/ | Å |) |
|---|-------|---|---|
|---|-------|---|---|

| | Plane 1 | | Plan | e 2 | | Plane | e 3 | Plane 4 | | |
|-----------------|----------|------------|------------------------------|-----------------|------|--------------|----------|----------|----------|--|
| | Sr | Ca | Sr | Ca | S | Sr | Ca | Sr | Ca | |
| | tartrate | tartrate | tartrate | tartrate | tart | rate | tartrate | tartrate | tartrate | |
| O(1) | | | -0.02 | -0.14 0.07 | | | | -0.08 | -0.02 | |
| O(2) | | | 0.06 | -0.17 | | | | 0.05 | -0.21 | |
| O(4) | | | 0.00 | • • • | Q |)∙05 | -0.01 | | | |
| O(5) | | | | | -0 |).09 | 0.004 | | | |
| O(6) | | | | | (|)•09 | -0.01 | | | |
| C(1) | 0.07 | -0.01 | -0.01 | 0.01 | | | | 0.08 | -0.05 | |
| $\mathbf{C}(2)$ | -0.06 | -0.01 | -0.01 | 0.21 | | | | 0.01 | 0.22 | |
| cà | -0.07 | 0.01 | | | -0 |)•07 | 0.01 | | | |
| C(A) | 0.06 | 0.01 | | | C |)•0 | 0.001 | | | |
| Sr | 0.00 | 0.01 | | | | | | 0.02 | 0.12 | |
| Ca | | + 101 | C(1) | $C(\mathbf{a})$ | C(2) | C(A) | | | 0.17 | |
| | | * Pl Pl | ane 1 $O(1)$ ane 2 $O(1)$ | O(2) | O(3) | C(4) C(1) | C(2) | | | |
| | | Pla | ane 3 O(4) | O(5) | O(6) | C(3) | C(4) | | | |
| | | Pl | ane 4 $O(1)$ | O(3) | C(1) | C(2) | Sr(Ca) | | | |

boxyl group of a different molecule. Of these, O(3A)-O(2) appears to be a non-bonded short contact in view of the large value (174°) assumed by the angle C(2A)-O(3A)-O(2).

A second network consists of hydrogen bonds formed through the water molecules. Two carboxyl oxygen atoms O(2A) and O(5A) belonging to the same molecule are linked by hydrogen bonds formed through the water molecules O(7A) and O(9), a pattern similar to that found in the crystal structure of Rochelle salt (Beevers & Hughes, 1941). The environments of the water molecules are shown in Fig.6. The water molecule O(7A) has three neighbours O(9), O(5A) and Sr forming a distorted trigonal arrangement. O(9) is similarly surrounded by Sr, O(7A) and O(2A) at distances 2.54, 2.78 and 2.45 Å respectively. The third water molecule is somewhat isolated. It donates a proton to O(2B). Instances of water molecules involved in a single hydrogen bond are not rare in crystal structures (the crystal structure of calcium thymidilate (Trueblood, Horn & Luzzati, 1961) is one such example). The distances and angles involved in hydrogen bonding and the non-bonded intermolecular distances are listed in Tables 8(a) and 8(b) respectively.

| Table | 8(<i>a</i>). | Dist | tances | and | angles | invol | lved | in l | hyd | rogen |
|-------|----------------|------|---------|-------|----------|---------|------|------|-----|-------|
| | bond | ling | in stro | ntiun | ı tartra | ate, ti | rihy | drat | е | |

| | Distance | Angle subtended at the water molecule |
|---|----------------------------|---|
| $\begin{array}{c} O(7A) \cdots O(9) \\ O(7A) \cdots O(5A) \end{array}$ | 2·78 Å 2·77 | 98° |
| $O(8) \cdots O(2A)$ | 2.63 | |
| $\begin{array}{l} O(9) \cdots O(2A) \\ O(9) \cdots O(7A) \end{array}$ | 2·45 2·78 | 94° |
| $O(4) \cdots O(6A)$ $O(3A) \cdots O(2)$ $O(3A) \cdots O(1)$ | 2·77 2·78 2·60 | |
| $C(2A)-O(3A)\cdots O(2)$ $C(2A)-O(3)\cdots O(1)$ $C(3A)-O(4A)\cdots O(6)$ | Angle 174° 126 96 | |

Table 8(b). Non-bonded intermolecular distances less than 3.5 Å in the structure of strontium tartrate

| For A, B refer to | Fig. 5 |
|---------------------|--------|
| $C(1A) \cdots O(8)$ | 3·07 Å |
| $C(2A) \cdots O(8)$ | 3.42 |
| $C(1) \cdots O(3A)$ | 3.00 |
| $C(4) \cdots O(4A)$ | 3.47 |
| $O(9) \cdots O(8)$ | 3.18 |
| $O(8) \cdots O(4)$ | 3.37 |
| $O(1A) \cdots O(4)$ | 3.21 |
| $O(3) \cdots O(6A)$ | 3.25 |
| $O(6) \cdots O(2A)$ | 3.19 |
| $C(1) \cdots O(6A)$ | 3.33 |
| $C(3) \cdots O(6A)$ | 3.27 |
| $O(8) \cdots O(5A)$ | 3.35 |







Fig.4. The environments of the carboxyl groups of calcium tartrate viewed down C(1)-C(2) and C(4)-C(3).



Fig. 5. The crystal structure of strontium tartrate trihydrate viewed down the b axis.

Molecular packing and hydrogen bonding: calcium tartrate tetrahydrate

The tartrate molecules lie in channels bonded by the cations, in a direction nearly parallel to the a axis, with the carbon atoms forming sheets parallel to the ab plane (Fig. 7). The structure is stabilized by a system



Fig.6. The environments of the water molecules in the structure of strontium tartrate.



Fig. 7. The crystal structure of calcium tartrate tetrahydrate viewed down the c axis.

of hydrogen bonds linking the molecules directly and also through the water molecules. The environments of the water molecules are shown in Fig.8. The water molecule O(10) has three neighbours O(4), O(3) and O(5A) situated at distances less than 3 Å. The angle O(4)–O(10)–O(3) is about 59° and it is likely that one of the protons of this water molecule is involved in the formation of a bifurcated hydrogen bond. The cation is situated close to the bisector of the lone pairs of orbitals of this water. A tentative scheme of proton assignment and also the bond distances and angles involved in hydrogen bonding are given in Table 9(a), and non-bonded intermolecular distances are listed in Table 9(b).

| Table | 9(<i>a</i>). | Dis | tances | and | angles | involved | in | hydrogen |
|-------|----------------|------|---------|-----|----------|----------|------|----------|
| | bond | ling | in calc | ium | tartrate | tetrahya | lrai | te |

| Donor (OD) | Acceptor (OA) | Distance $(OD) \cdots (OA)$ | Angle subtended at the water molecule |
|---------------|------------------|-----------------------------|---|
| O(7) | O(8) | 2·72 Å | 123° |
| O(7) | O(6C) | 2.91 | |
| O(8) | O(6A) | 2.94 | 130 |
| O(8) | O(2) | 3.03 | |
| O(9) | O(8) | 3.04 | 137 |
| O(9) | O(4C) | 2.72 | |
| O(10) | O(3) | 2.99 | 109 |
| O(10) | O(5A) | 2.87 | |
| O(4) | O(10) | 2.95 | |
| O(3) | O(1B) | 2.58 | |
| Other angle | es involved in h | ydrogen bonding | |
| C(3) - O(4) | ····O(9) | 105° | |
| C(2)-O(3) | $\cdots O(1B)$ | 104 | |

| $\mathcal{O}(\mathcal{I}) = \mathcal{O}(\mathcal{I})$ | 105 |
|---|-----|
| $C(2)-O(3)\cdots O(1B)$ | 104 |
| $C(6) - O(8) \cdots O(7)$ | 141 |
| $O(3) - O(10) \cdots O(4)$ | 60 |
| $O(4) - O(10) \cdots O(5)$ | 98 |
| | |

| Table $9(b)$. | Non-bonded | intermolecular | distances less |
|----------------|---------------|------------------|----------------|
| than 3.5 | Å in the stra | ucture of calciu | m tartrate |

For A, B, C refer to Fig. 7.

| $O(7) \cdots C(3A)$ | 3·32 Å | $O(3) \cdots O(2B)$ | 3·13 Å |
|----------------------|--------------|----------------------|--------|
| $O(7) \cdots O(2A)$ | 3.10 | $O(2B) \cdots O(9)$ | 3.48 |
| $O(10) \cdots O(2A)$ | 3.19 | $O(1B) \cdots O(8)$ | 3.14 |
| $O(6A) \cdots O(5)$ | 3.02 | $O(1B) \cdots O(9)$ | 3.44 |
| $O(8) \cdots O(5A)$ | 3.35 | $O(7B) \cdots O(10)$ | 3.32 |
| $O(9B) \cdots O(5C)$ | 3.36 | $O(9B) \cdots O(10)$ | 3.18 |
| $C(1B) \cdots O(3)$ | 3.21 | $O(9) \cdots C(3C)$ | 3.39 |
| $C(1B) \cdots O(9)$ | 3.30 | $O(10) \cdots C(3C)$ | 3.43 |
| $C(2B) \cdots O(1)$ | 3.25 | $O(7) \cdots C(4C)$ | 3.47 |
| $C(4) \cdots O(1C)$ | 3.45 | $O(9) \cdots O(2C)$ | 3.22 |
| $O(3C) \cdots O(4)$ | 3.26 | $O(5A) \cdots O(1B)$ | 3.08 |
| $O(3C) \cdots O(6)$ | 3 ∙48 | | |
| | | | |

Coordination of strontium and calcium

In both compounds the cations exhibit 8-fold coordination. The coordination polyhedra in both cases are distorted dodecahedra (Figs. 9 and 10). The cationoxygen distances are given in Table 10. The average Sr-O distance is 2.65 Å which agrees with the sum of the ionic radii of Sr²⁺ and O²⁻ (2.53 Å). The average Ca²⁺-O distance observed in this crystal is 2.47 Å, which agrees with the values reported in other examples of 8-coordinate calcium (2.49 Å in CaH(PO₄), 2.52 Å in Ca(H₂PO₄). H₂O; MacLennan & Beevers, 1955, 1956).

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| Table 10. | Cation-oxygen | distances | in the | structures | of |
|-----------|---------------|-----------|---------|------------|----|
| | strontium and | calcium t | artrate | S | |

| Sr -O(4) O(5 <i>A</i>) O(2 <i>A</i>) O(3 <i>A</i>) O(7 <i>A</i>) | 2·55 Å 2·62 2·54 2·58 2·57 |
|--|--|
| O(8A) O(1A) | 2·60 2·60 |
| O(9) | 2.54 |
| Ca-O(1) O(3) | 2·43 A 2·54 2·52 |
| O(2B) O(5B) O(8B) | 2·42 2·47 |
| O(4 <i>C</i>) O(10 <i>C</i>) O(6 <i>C</i>) | 2·51 2·48 2·39 |
| 0(00) | 2 37 |



Fig.9. The coordination polyhedron of strontium.



Fig.8. The environments of the water molecules in the structure of calcium tartrate.



Fig. 10. The coordination polyhedron of calcium.

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